

# Mean-link tadpole improvement of SW and D234 actions

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We investigate a tadpole-improved tree-level- $\mathcal{O}(a^3)$ -accurate action, D234c, on coarse lattices, using the mean link in Landau gauge to measure the tadpole contribution. We find that D234c shows much better rotational invariance than Sheikholeslami-Wohlert, and that mean-link tadpole improvement gives much better hadron mass scaling than plaquette tadpole improvement, with finite- $a$  errors at 0.25 fm of only a few percent.

We explore the effects of possible  $\mathcal{O}(\alpha_s)$  changes to the improvement coefficients, and find that the two leading coefficients can be independently tuned: hadron masses are only sensitive to the clover coefficient  $C_F$ , while hadron dispersion relations are only sensitive to the third derivative coefficient  $C_3$ . Preliminary non-perturbative tuning of these coefficients yields values that are consistent with the expected size of perturbative corrections.

## 1. Introduction

Because the cost of lattice QCD calculations is so sensitive to the lattice spacing, improved actions on coarse lattices are an attractive approach. In this paper we study a tadpole-improved (TI) tree-level  $\mathcal{O}(a^3)$ -accurate action, D234c, comparing its predictions with those of the Sheikholeslami-Wohlert (SW) tadpole-improved tree-level  $\mathcal{O}(a)$ -accurate action [1]. Since we are interested in finite- $a$  errors, we use quark masses near the strange quark mass, thereby avoiding the separate complications of chiral extrapolation.

We use the mean link in Landau gauge rather than the traditional plaquette prescription for calculating our TI factor  $u_0$ . This is known to give smaller scaling errors in the NRQCD charmonium hyperfine splitting [2], and to give a clover coefficient for Wilson glue that agrees more closely [3] with the non-perturbatively determined value [4].

Our glue action is a tree-level tadpole-improved plaquette and rectangle action [5,6],

$$S = -\beta \sum_{x,\mu} \left\{ \frac{5}{3} \frac{P_{\mu\nu}(x)}{u_0^4} - \frac{R_{\mu\nu}(x) + R_{\nu\mu}(x)}{12u_0^6} \right\} \quad (1)$$

where  $P_{\mu\nu}$  and  $R_{\mu\nu}$  are the plaquette and  $2 \times 1$

rectangle.

For hadron spectrum measurements we generated gluon configurations at two lattice spacings, 0.40 fm and 0.25 fm, using lattices of the same physical size (2 fm) at both lattice spacings.

The D234c Dirac operator [7] is

$$M = m_c \left( 1 + \frac{ram_c}{2} \right) + \sum_{\mu} \left\{ \gamma_{\mu} \Delta_{\mu} - \frac{C_3}{6} a^2 \gamma_{\mu} \Delta_{\mu}^{(3)} + r \left[ -\frac{1}{2} a \Delta_{\mu}^{(2)} - \frac{C_F}{4} a \sum_{\nu} \sigma_{\mu\nu} F_{\mu\nu} + \frac{C_4}{24} a^3 \Delta_{\mu}^{(4)} \right] \right\}.$$

with  $r = C_F = C_3 = C_4 = 1$  at tree level.  $\Delta_{\mu}^{(n)}$  is a tadpole-improved lattice discretization of the gauge-covariant  $n$ 'th derivative, and  $F_{\mu\nu}$  is the improved field strength [7].

## 2. Hadron mass scaling

In Figure 1 we show that mean-link tadpole improvement leads to much smaller finite- $a$  errors for the vector meson mass. (We fixed our lattice spacing relative to SCRI's by the "Galilean quarkonium" method [8].) The remaining finite- $a$  errors are due to a mixture of radiative corrections to the tree-level coupling constants, and higher-order interactions not included in the D234c action.

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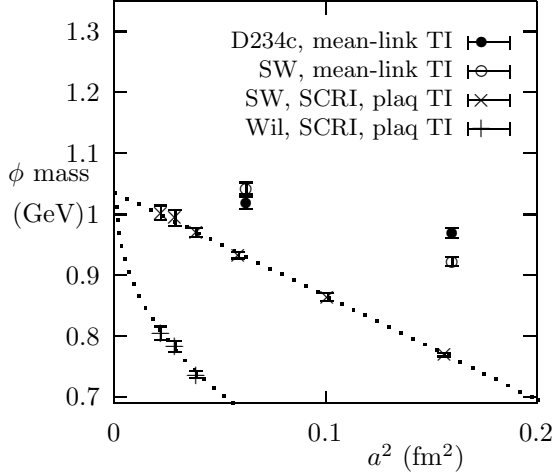


Figure 1. Mass of  $\phi$  meson as a function of lattice spacing,  $P/V = 0.70$ . Mean link TI ( $\bullet$  D234,  $\circ$  SW) is more continuum-like than plaquette TI ( $\times$  SW,  $+$  Wilson, data supplied by SCRI)

To investigate the sensitivity of the hadron masses to radiative corrections we measured the fractional change caused by multiplying each coupling constant in turn by  $1 + \alpha_s$ , (Table 1). We see that the only coupling for which radiative corrections are important is the clover coupling,  $C_F$  [9]. Mean-link tadpole improvement increases the clover coefficient, and reduces finite- $a$  errors in the vector mass for both D234c and SW, as seen in Figure 1. The superiority of D234c will become apparent when observables sensitive to Lorentz violation are considered (see below).

We performed a quadratic  $a \rightarrow 0$  extrapolation of SCRI's SW results to estimate the continuum  $V$  mass. Adjusting the clover coefficient to give this  $V$  mass at  $a = 0.25$  fm, we find that choosing

$$C_F = 1 + \alpha_s/6, \quad (2)$$

where  $\alpha_s \equiv 3/2\pi\beta$  is the bare coupling, reduces the  $V$  mass error at 0.25 fm from 2(1)% to 0(1)%, and also at 0.4 fm from 7(1)% to 0(1)%. This suggests that perturbative corrections to  $C_F$  are relatively small after tadpole improvement. (The quadratic fit of SCRI's SW data may be too naive:

the true continuum value could be different by a few percent, changing the coefficient of  $\alpha_s$ , but leaving it perturbative in size.)

A perturbative analysis of  $C_F$  through  $\mathcal{O}(\alpha_s)$  will soon be completed [10], and will be compared with this nonperturbative estimate. (Tuning of  $C_F$  using PCAC is another possibility [11].) Whatever the result of this comparison, the non-perturbative formula (2) should give hadron masses accurate to a few percent for lattice spacings up to 0.4 fm.

	$C_F$		$C_3$	$C_4$
$a$ (fm)	0.25	0.4	0.4	0.4
P	11(1)	35(1)	0.9(1)	-3.2(1)
V	10(1)	35(1)	1.0(2)	-3.0(3)
N	6(2)	26(2)	0.9(3)	-2.7(4)
D	8(1)	25(2)	1.0(1)	-2.1(4)

Table 1

Percentage change in hadron masses at  $P/V \approx 0.76$  when individual coefficients in the mean-link tree-level TI D234c quark action are multiplied by  $1 + \alpha_s$ .

### 3. Lorentz Violations

From Figure 1 the SW action appears to work well even at 0.4 fm, however it seriously violates Lorentz invariance. To see this, consider the speed of light

$$c^2(\mathbf{p}) = \frac{E^2(\mathbf{p}) - E^2(0)}{\mathbf{p}^2} \quad (3)$$

which should equal 1, for all  $\mathbf{p}$ . This quantity is particularly sensitive to the  $C_3$  term in the D234c action since this term cancels the leading Lorentz-violating error. Our results for  $c^2$ , for both pseudoscalar and vector mesons, are shown in Figure 2. At 0.4 fm, D234c is dramatically superior: it deviates from  $c^2 = 1$  by only 3–5% at zero momentum, and by less than 10% even at momenta of order  $1.5/a$ , while SW gives results

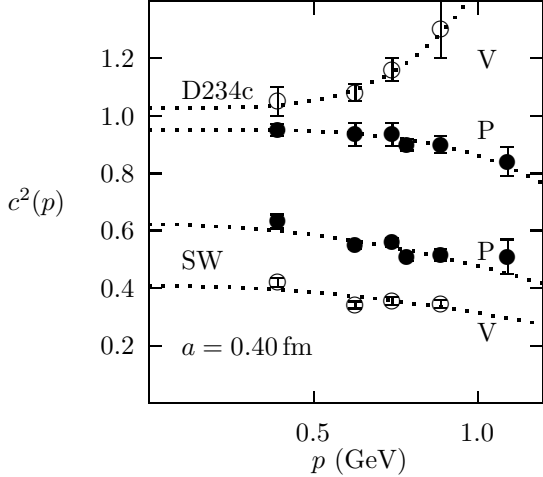


Figure 2. Comparison of speed of light squared (3) for D234c and SW actions. Pseudoscalar (P) and Vector (V) mesons at  $P/V \approx 0.7$  are shown.

that deviate by 40–60% or more for all momenta, including zero. The reason for SW’s poor performance is that the strange quark is relatively massive at our lattice spacings: the  $\phi$  mass is  $2.1/a$  at  $a = 0.4$  fm. These results illustrate that D234c is far more accurate for hadrons with large masses.

The  $C_3$  term is the only  $a^2$  correction that breaks Lorentz invariance, so we can use  $c^2$  to tune  $C_3$  nonperturbatively. At 0.4 fm we tuned  $C_3$  to make the dispersion relation for the lightest meson, the pseudoscalar, perfect at low momentum: when

$$C_3 = 1.2 \approx 1 + \alpha_s/2 \quad (4)$$

we obtain the  $c^2(p=0)$  results shown in table 2. The pseudoscalar (P) moves to within  $\pm 2\%$  of  $c^2 = 1$ . (The vector (V) shows 20% deviations, because it is 25% heavier than the P at this quark mass; lowering the quark mass to  $P/V = 0.6$  we see that, within errors, both P and V show no Lorentz violation at low momentum.)

As with  $C_F$ , the non-perturbative estimate of the quantum corrections to the mean-link TI tree-level value of  $C_3$  indicates that they are perturbative in size, and a perturbative calculation of  $C_3$  is currently in progress.

	$P/V \approx 0.76$		$P/V \approx 0.6$
	$C_3 = 1.0$	$C_3 = 1.2$	$C_3 = 1.2$
P	0.95(2)	1.00(1)	0.99(2)
V	1.02(4)	1.21(5)	1.04(3)

Table 2

$c^2(\mathbf{p})$  extrapolated to  $p = 0$ . Tuning P to  $c^2 = 1$  gives  $C_3 = 1.2$ . Only at lower quark mass ( $P/V = 0.6$ ) can both P and V be tuned simultaneously.

#### 4. Conclusions

The D234c action is a very useful tool for lattice calculations involving large quark masses or momenta, making it particularly suitable for coarse lattices, high-momentum form factors, heavy quarks, and finite temperature studies. With more precise (perturbative or non-perturbative) tuning of the leading coefficients it should be accurate to within a few percent at lattice spacings as large as 0.4 fm and meson masses as large as  $2/a \approx 1$  GeV. We are also continuing our previous work with D234 on anisotropic lattices, which will push the range of accessible energies even higher.

We acknowledge the Cornell Theory Center for the use of its SP-2 computer in this research. MGA supported by DOE DE-FG02-90ER40542.

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